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The Great Industry Gamble: Market Structure Dynamics as a Survival Contest

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Abstract

Industry dynamics are studied as an endogenous tournament with infinite horizon and stochastic entry. In each period, firms' investments determine their probability of surviving into the next period. This generates a survival contest, which fuels market structure dynamics, while the evolution of market structure constantly redefines the contest. More concentrated markets endogenously generate less profit, rivals that are more difficult to outlive, and more entry. The unique steady state distribution exhibits ongoing turbulence, correlated exit and entry rates and shake-outs. The model's predictions fit empirical findings in markets where firms trade off profits for smaller risk of failure (e.g. banking).

JEL: D43, L10, L13

Keywords: Industry Dynamics, Contest, Reputation, Banking

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1 Introduction

Market competition is often viewed as a survival contest. Risky investment decisions (e.g. Lehman Brothers, LTCM), technological setbacks (e.g. Polaroid, Commodore Computers), fraud, corporate scandals and lost reputation (e.g. Barings Bank, Arthur Andersen) can wipe out well-established companies, sometimes in a matter of months. In many instances, a firm faces a trade off and can increase profits at the expense of risking failure. For instance, a financial institution can increase returns by investing in riskier assets, which in turn increases its probability of insolvency. Or a firm can decrease cost by lowering unobserved quality while risking its reputation and ultimately its business. Also, a manufacturer can reduce R&D expenditure while jeopardising vital innovation that secures future market presence. Firms take calculated risks in a strategic space when they trade off the costs and benefits of survival. Survival can pay not only because current profits can be earned again tomorrow, but also because the failure of rivals may change competitive conditions and increase future profits. In this study, market structure is modelled as a dynamic tournament, where the main driving force is strategic gambling: a firm gambles on its survival, knowing that rivals do just the same.

Allowing for the full spectrum of market conduct, I develop a simple oligopoly model and analyse the interaction between firms' survival strategies and market structure in a dynamic stochastic game. In particular, a firm invests (or exerts costly effort) in every period and this investment determines the probability that it survives into the next period. The rewards to investment are realised in the future and depend not only on how such investment affects the firm's probability of survival, but also on how many other firms are likely to be competing with the firm in the future. The latter in turn depends on present and future rivals' survival strategies as well as entry. The result is a contest, in which firms try to outlive each other.

In this model, investments in survival strictly increase with market concentration in equilibrium, despite the fact that concentration frustrates the surviving firms in three fundamental ways. To see these, consider the case of perfect Bertrand competition, where only a monopoly generates positive revenues, and hence the only reason for a firm to be in the market is the hope

that one day it may become a monopolist for some time. First, in this case profits are lower in more concentrated markets as investment increases. Second, rivals also invest more in concentrated markets and hence are more difficult to outlast. Consequently, the market tends to get stuck in concentrated states close to, but without reaching, the prize (i.e. monopoly). Third, market concentration endogenously generates more entry, which sets back the market into less concentrated states, thereby naturally eroding surviving incumbents' hard fought position in the contest. In sum, as the market converges towards concentration, a reduction (i.e. fewer firms) is counteracted by an increase in rivalry (i.e. rivals investing more in survival and more entry). In equilibrium, the former effect always dominates the latter, which results in higher firms' net present values, and consequently higher investments, in more concentrated markets. The market exhibits ongoing turbulence and the steady state distribution of market structures is unique and follows a lognormal pattern.

The game is a natural description, for instance, of markets with severe moral hazard problems, where loss of reputation can lead to sudden failure of a firm.¹ Consider the workhorse of the reputation literature, the standard unobserved quality framework, where consumers don't observe quality before purchase. In a pure moral hazard version of the model (e.g. Allen 1984, Rob and Fishman 2005), firms exert costly effort in each period, thereby increasing the probability of producing a high quality product at the end of the period. If consumers do not tolerate anything but high quality in equilibrium, then exerting effort is essentially an investment in the probability of survival. I discuss this extended game in some detail in Appendix A and as such sketch a possible microfoundation of the reduced form model discussed in the body of the article.² The present model is a more abstract description of markets, where the fate of a firm

¹Examples that spring to mind are the demise of Arthur Andersen, the long history of runs in the financial industry, and most recently the sudden and almost complete disappearance of the market for new securitization as a result of Credit Rating Agencies' (CRA) previous rating practices in structured finance. These can all be taken as natural examples of firm failures due to lost reputation.

²In the reputation literature, unobserved quality has been investigated extensively in perfect competition (e.g. Klein and Leffler 1981 or Hörner 2002) or local monopoly settings (Rob and Fishman 2005). A handful of studies extend the scope of analysis to oligopoly with static market structure (e.g. Allen 1984, Bar-Isaac 2005 and Dana and Fong, forthcoming). However, to the best of my knowledge, this is the first study that provides an analysis of the dynamic interplay between firms' unobserved quality choice and the endogenous market structure.

plays out in the longer run and hence firm failure is typically the result of slow decay. However, even in these markets the contest for survival is perhaps a reasonable concept if the stage game is thought of as spanning a longer horizon.

The literature on contests is vast, see Konrad (2009) for a recent survey. The survival contest analysed in the present model is quite different from previous frameworks. First, tournaments are usually finite by nature, the game ends when an agent wins a fixed prize. In my model, however, the prize(s) can be won and then lost and won again over an infinite horizon. This feature is similar to e.g. Budd et al (1993), Aghion et al (2001), Cabral (2002) and Hörner (2004). These duopoly models focus on the strategic effects of asymmetry between leader and laggard. My model sacrifices the analysis of asymmetry in order to develop an analytically tractable oligopoly model where traditional questions on market structure, such as the effect of entry, can be investigated. Second, the structure of the tournament is endogenous. Elimination tournaments are (finite) dynamic games comprised of a series of subcontests, where the structure of subcontests and the process from one subcontest to the other are fixed (e.g. only the winners of quarter finals can proceed to the semi finals). (e.g. Rosen 1986, Moldovanu and Sela 2006, Groh et al 2012) In my model, however, the number of winners in a subcontest is not fixed; indeed, in principle all players (or none) can survive and proceed to the next contest. Depending on firms' strategies, some subcontests (i.e. market structures) are more likely to be reached than others. This generates a steady state distribution of subcontests. Third, the prize is endogenous as the number and the actual values of these prizes depend on the strength of competition that the contest endogenously generates.

From a technical point of view, the literature on industry dynamics can be divided into four major strands. The earliest literature investigates market structure dynamics typically in perfect competition settings (e.g. Jovanovic 1982, Hopenhayn 1992). The second strand incorporates strategic interaction into the analysis in duopoly models (e.g. Maskin and Tirole 1988, Cabral and Riordan 1994). The third line of investigation develops a very general game theoretic framework allowing for, inter alia, an arbitrary number of firms and firm heterogeneity in a rich stochastic structure (see Ericson and Pakes 1995 and the survey of the subsequent

literature by Doraszelski and Pakes 2007). The primary motivation of this design is to produce a framework for empirical work; hence, these models are typically suitable only for numerical analysis and deliver “very little in the way of analytical results of applied interest; i.e. just about anything can happen.” (Doraszelski and Pakes 2007). Lastly, the fourth strand retains the game theoretic setting but keeps it analytically tractable, while also allows for arbitrary number of firms and some exogenous stochastic structure (Amir and Lambson 2003). My model is in this latter tradition.

I study a framework where the dynamics of firm failures and entry are completely endogenous and are not driven by exogenous shocks. The only source of uncertainty external to the firm is rivals’ entry costs, the distribution of which is fixed over time. Market conditions (i.e. prices, technology, etc) are deterministic at the outset and hence the stochastic evolution of the market is solely governed by firms’ survival strategies and the entry that ensuing failures generate. I introduce stochastic entry in a novel way. Although stochastic entry is the norm rather than the exception, the number of entrants often doesn’t affect the underlying stochastic structure of previous models (e.g. Jovanovic 1982, Hopenhayn 1992, Amir and Lambson 2003). In other words, entry is usually only stochastic because the state evolves stochastically but in equilibrium entry is a deterministic function of the state. I depart from the literature by allowing for an analytically tractable entry process whereby equilibrium entry is endogenous and a stochastic function of the state of the game.

The article is organized as follows. In Section 2, I outline the baseline model. I extend that with the introduction of stochastic entry in Section 3. The main features of the model are discussed in Section 4. An example of an industry (banking) where empirical findings are consistent with the model’s predictions is provided in Section 5. I conclude in Section 6. Appendix A contains an extension of the model where I present a possible microfoundation of the reduced form game discussed in the body of the article. All the lengthy proofs are relegated to Appendix B.

2 The baseline model

Time is discrete and infinite, firms discount the future with a common factor $\beta \in (0, 1]$. The number of firms present in the market is denoted by $n \in \{0, 1, 2, \dots\}$. In each period, firms engage in price or quantity competition and choose their probability of survival into the next period. For simplicity, I do not model explicitly the price/quantity game (see Appendix A for an example). The symmetric equilibrium profit for a firm from the price/quantity game (henceforth “market game”) is denoted by $\pi(n)$.

Assumption 1. $\pi(n) \leq \pi(n - 1)$ for $n > 2$, $\pi(2) < \pi(1)$, $0 \leq \pi(\cdot) < \infty$, and $\lim_{n \rightarrow \infty} \pi(n) = 0$.

The per-period gross profit $\pi(n)$ is non-negative and weakly decreasing in the number of firms, the monopoly profit is strictly positive. Assumption 1 conforms with standard models of homogeneous as well as differentiated product (Bertrand or Cournot) competition and hence this specification essentially captures all the cases typically considered in the literature.³ In each period, each firm $i = 1, \dots, n$ chooses its probability of survival $x_i \in [0, 1]$ at a cost of $g(x_i)$ and hence its per-period net profit is $\pi(n) - g(x_i)$. The cost function can be interpreted in several ways. For instance, in a standard unobserved quality framework, where unforgiving consumers do not tolerate low quality in equilibrium, firms exert costly effort to increase the probability of producing high quality. This example is explored in more detail in Appendix A. However, note that $g(x_i)$ need not represent actual costs, it can also be thought of as returns that the firm forgoes in order to reduce business risk. In this case, $\pi(n)$ is potential profit that the firm realises if it chooses the riskiest business strategy (i.e. $x_i = 0$) and $g(x_i)$ is the profit sacrificed to increase the probability of survival. For instance, financial institutions sacrifice higher returns when they embark on a less risky investment strategy. Also, auditors and credit rating agencies (CRA) give up fees to preserve reputation when they give an unfavourable audit or rating to a client who may then choose to switch. The fewer dubious companies it yields to and the more potential fees it sacrifices, the less probable it is that a scandal will bankrupt the

³For instance, in a Cournot setting both quasi-competitive and quasi-anticompetitive games yield weakly decreasing per-firm profits. (Amir and Lambson 2000)

auditor or CRA in the next period. I make the following assumption on the cost function.

Assumption 2A. $g : [0, 1] \rightarrow [0, g(1)]$, $g(x_i), g'(x_i), g''(x_i) > 0$ for $x_i > 0$ and $g(0) = g'(0) = 0$, $g(1) < \infty$.

The cost function is strictly increasing and convex. The assumption of bounded costs is not essential; the main results are unaffected if I relax it.⁴ Its only purpose is to allow for a corner solution. Without entry, this is one simple way to ensure that the market structure converges to a steady state. Were the cost to tend to infinity, $\lim_{x_i \rightarrow 1} g(x_i) = \infty$, then not even a monopoly would find it optimal to invest the maximum amount ($x_i = 1$) with finite gross profits. As a consequence, even a monopoly would fail sooner or later and the market would vanish in the long run. With entry, however, we do not need the assumption of bounded costs in order for the market structure to converge to a steady state, as shown in Section 3.

Including firm i , the number of firms (the market structure) at the beginning of the next period will be $n - k$ with probability $x_i \Pr(k|x_{-i})$, where $x_{-i} = [x_j]_{j \neq i}$ and $k = 0, \dots, n - 1$ is the number of firms other than firm i failing and exiting the market at the end of the period.⁵ $\Pr(k|x_{-i})$ is the probability mass function of the convolution of $n - 1$ Bernoulli distributions with success probabilities x_{-i} . For expositional purposes, I first present the model without entry.

Assumption 3A. *There is no entry.*

Assumption 1, 2A and 3A are maintained throughout this section. The stage game is infinitely repeated, the strategies are assumed to be Markov and I focus on symmetric Markov

⁴Bounded costs are reasonable. Consider, for instance, banks competing for deposits. In the market game, each bank acquires a symmetric share of the deposits at the equilibrium deposit rate, which will be invested in risky assets. The bank can ensure a profit $\pi(n)$ if it invests in the riskiest asset available (i.e. $x_i = 0$), but it can also increase the probability of survival by sacrificing $g(x_i)$ of this potential profit if it invests in assets with lower risk and return. The net profit $\pi(n) - g(x_i)$ can even be negative if the returns on safe assets are lower than the equilibrium deposit rates. If the risk (i.e. return on assets) is bounded, so is the potential revenue that a bank can sacrifice in order to secure survival.

⁵Firm failure need not be interpreted literally. In the presence of entry, failure may just mean that the firm needs to incur some additional ("entry") cost if it wishes to compete in the market again. For instance, loss of reputation can lead to some cost of rebranding (e.g. litigation costs) - a sunk cost necessary to "re-enter" the market. Also, in a game where innovation in each period is necessary to maintain market presence, unsuccessful innovators can again enter the market next period by e.g. buying patents.

Perfect Equilibrium (MPE) throughout the analysis.⁶ That is, strategies depend only on payoff relevant information. The payoff relevant information can be conveniently condensed into a state variable, which is the number of firms n .

With n firms in the market, the dynamic programme of firm i is simply

$$v(n; x_{-i}) = \pi(n) + \max_{0 \leq x_i \leq 1} \left\{ -g(x_i) + \beta x_i \sum_{k=0}^{n-1} V(n-k) \Pr(k|x_{-i}) \right\} \quad (1)$$

where n is the current state and $V(n-k)$ is the value of the firm in a symmetric equilibrium in state $n-k$. After differentiating and imposing symmetry (i.e. $x_i = x$ for all i), the first order conditions (FOC) of this programme are

$$\beta \sum_{k=0}^{n-1} V(n-k) \binom{n-1}{k} (1-x)^k x^{n-k-1} - g'(x) + \zeta - \varrho = 0, \quad \zeta x = 0, \quad \varrho(x-1) = 0 \quad (2)$$

where ζ, ϱ are Lagrange multipliers, and I use the fact that if $x_j = x$ for all $j \neq i$, then $\Pr(k|x_{-i}) = \binom{n-1}{k} (1-x)^k x^{n-k-1}$. The simple nature of the first order conditions is the result of two facts. First, the convolution of Bernoulli trials is simply the binomial distribution, due to symmetry. Second, the Bernoulli probabilities are linear. The latter may appear overly simplistic. However, it is not more restrictive than assuming non-linear survival probabilities but constant marginal costs, a commonly applied modelling structure (e.g. Rob and Fishman 2005, Ericson and Pakes 1995). In the present model the marginal benefit is constant (given rivals' strategy) and the marginal cost is not, whilst in the other models it is exactly the opposite. In both cases the trade off between a unit cost and a unit benefit is non-linear and the first order conditions yield non-linear strategies in the state variable.

⁶Although the symmetry of equilibrium strategies is certainly a major limitation of the analysis, there is some evidence that firms' risk taking strategies are similar: for instance, Eisenberg and Macey (2004) finds that the quality of Arthur Andersen audits was no different from other major accounting firms. Laia and Gula (2008) arrive at the same conclusion for the case of Laventhol and Horwath, the seventh biggest auditor, which declared bankruptcy in 1990. Similarly, banks tend to get into trouble together in ways that contagion can only partially explain.

The solution to the first order conditions (2) is denoted by $x(n)$. Given Assumptions 1 and 2, it is easy to see that $x(n) \neq 0$. Intuitively, $x(n) = 0$ cannot be a symmetric equilibrium because when all rivals fail with probability one, the firm will be a monopolist next period and hence it has an incentive to invest in survival. There are two possible solutions: one corner and one interior. It is $x(n) = 1$ as long as

$$\pi(n) \geq g'(1) \frac{1-\beta}{\beta} + g(1) \quad (3)$$

Alternatively, $0 < x(n) < 1$ where $x(n)$ solves (2) with $\zeta = \varrho = 0$.

If inequality (3) holds, firms invest the maximum amount, there is no gambling and hence no failure. The necessary condition is that firms generate enough revenue from the market game to cover the cost. If firms do not discount the future ($\beta = 1$), then non-negative net profits ($\pi(n) - g(1) \geq 0$) are also sufficient for the market structure being at a standstill. However, as firms' impatience grows ($\beta \rightarrow 0$), the incentive to survive decreases and firms gamble, regardless of profits.

I first establish the existence of a symmetric pure strategy equilibrium.

Proposition 1. *In the game without entry, there exists a symmetric Markov Perfect Equilibrium in pure strategies.*

Proof. See Appendix B. □

Note from the proof that the reaction functions are linear, therefore the symmetric equilibrium is in fact unique as long as the slope of the reaction functions is not unity (i.e. $|\partial x_i / \partial x_j| \neq 1$). In other words, asymmetric equilibria may emerge only in a very special case. Also, investments in survival are strategic substitutes (i.e. $\partial x_i / \partial x_j < 0$). The next Proposition follows directly from the first order conditions:

Proposition 2. *In a symmetric equilibrium of the game without entry, in the long run the number of firms is no bigger than \tilde{n} , where*

$$\tilde{n} = \begin{cases} 0 & \text{if } \pi(1) < g'(1)^{\frac{1-\beta}{\beta}} + g(1) \\ \max\{n : \pi(n) \geq g'(1)^{\frac{1-\beta}{\beta}} + g(1)\} & \text{otherwise} \end{cases}$$

It is immediate from Proposition 2 that higher cost of investment and, perhaps more surprisingly, fiercer competition in the market game ($\pi(n)$ lower for all n) leads to more concentrated markets in the long run. Greater patience, as captured by the discount factor, supports the existence of more firms in a long run equilibrium.

The following Proposition states the main result in the absence of entry. It shows that equilibrium investment is strictly higher in more concentrated markets even when gross profits $\pi(n)$ do not increase with concentration. As a consequence, paradoxically firms in more concentrated markets can be *less* profitable.

Proposition 3. *In a symmetric equilibrium of the game without entry, more firms in the market implies strictly less investment in survival when $n > \tilde{n}$. That is, $x(n+1) < x(n) < 1$.*

Proof. See Appendix B. □

Importantly, note that it is sufficient that gross profits are only weakly decreasing with the number of firms. The intuition behind the result is a survival contest that the model dynamics generate independent of current profits. To see this, consider homogeneous-product Bertrand competition, where $\pi(n+1) = \pi(n) = 0$ for all $n > 1$. In this case, investment in survival is driven by future expectations of market structure, i.e. by the possibility that a firm may become a monopolist. Chances of winning this contest are remote in fragmented markets because too many firms compete for the prize. But as failing firms drive the market towards concentration, the stakes rise because there are fewer rivals to outlive; consequently, firms care more about the future and invest more. The increasing investment schedule, however, makes surviving firms

less profitable when gross profits $\pi(n)$ do not increase with concentration. Furthermore, rivals too invest more and fail with smaller probability. Note, therefore, the two countervailing effects at work: although failing rivals naturally increase a surviving firm's chance of winning the contest (i.e. increase its continuation payoff), surviving rivals are harder to outlive, which in turn upsets the prospects (i.e. decreases the continuation payoff). In other words, a reduction is counteracted by an increase in rivalry. In equilibrium, the former effect always dominates the latter.

The simple structure of the baseline model allows me to fully characterise the equilibria by comparative static analysis. The following Proposition summarizes these results.

Proposition 4. *In a symmetric equilibrium without entry, the higher the per-period gross profit $\pi(n)$, the monopoly profit $\pi(1)$, or the discount factor β , the more firms invest in survival. That is, $dx(n)/d\pi(n) > 0$, $dx(n)/d\pi(1) > 0$, $dx(n)/d\beta > 0$.*

Proof. See Appendix B. □

The effects of gross profits and the discount factor are intuitive. Perhaps more surprisingly, higher monopoly gross profit increases equilibrium investments, irrespective of the number of firms. This means that paradoxically the higher the monopoly value, the less probable that the market ever becomes a monopoly: as the value of monopoly tends to infinity, firms engage in fiercer competition to survive and with probability approaching one none will fail, preserving current market structure almost surely. The following corollary, which is a direct consequence of Proposition 4, summarises this observation.

Corollary 1. *In a symmetric equilibrium of the game without entry, as the value of the monopoly firm approaches infinity, the optimal investment in survival tends to the maximum value. That is, $\lim_{\pi(1) \rightarrow \infty} x(n) = 1$. As a consequence, the expected number of failures tends to zero for any n : $\lim_{\pi(1) \rightarrow \infty} n(1 - x(n)) = 0$.*

The survival contest interpretation of the game, the “contest for a prize”, has some features in common with patent races. In both models, the higher the prize, the more firms invest. However, there is at least one crucial difference between the current model and standard patent races. The survival contest identified in the current model is not a race, in fact it is quite the opposite. In a simple symmetric patent race more investment in the probability of discovery would imply a quicker end of the game because the probability that one of the firms wins is increasing with investments. In the current model, however, higher level of investment increases the probability of firm survivals and as such *reduces* the probability that any of the firms succeeds in winning the prize (e.g. becomes a monopolist in the Bertrand case). In other words, the probability that the prize will be won is decreasing with the value of the prize, which is in contrast to what we observe in patent race models.

3 Entry

Entry can be introduced into the model in several ways. In an earlier version of the article, I studied free and deterministic entry. This entry process leads to much simplified dynamics, however: market structure steadily converges to a state that is absorbing.⁷ I introduce stochastic entry in order to analyse more realistic processes. The entry process described below differs from existing literature because entry, despite being sequential, is not a deterministic function of the state in equilibrium.⁸ (see e.g. Jovanovic 1982, Hopenhayn 1992, and Amir and Lambson 2003)

To simplify the exposition, in what follows I will focus only on interior solutions in which $x_i \in (0, 1)$. For this purpose, Assumption 2A is replaced by the following modified assumption.

⁷In this simple setup, when there are too many firms in the market, there is no entry and the market converges to the state, where firm value is equal to the cost of entry. Once this state is reached, free entry immediately fills up any gap created by failing firms subsequently, so states with fewer firms are never reached in equilibrium.

⁸A sequential structure is employed for two reasons. First, it facilitates a very simple recursive formulation. Second, this is the traditional way of modelling entry in IO (see Doraszelski and Pakes 2007). However, for the steady state analysis, a continuous time approximation will be applied shortly where the differences between sequential and simultaneous entry disappear at the limit.

Assumption 2B. $g : [0, 1) \rightarrow [0, \infty]$, $g(x_i), g'(x_i), g''(x_i) > 0$ for $x_i > 0$ and $g(0) = g'(0) = 0$, $\lim_{x_i \rightarrow 1} g(x_i) = \infty$.

In this section, the stage game is composed of a sequence of $N + 1$ rounds. More precisely, in each period there are N entry rounds followed by one production round. In each entry round only one entrant may enter; in other words, in each period there are N potential entrants who enter sequentially. The last entry round is then followed by a production round, which is essentially the same as in the previous section: firms play the (unmodelled) market game and invest simultaneously in survival. The stage game is then infinitely repeated, as before. Time “stops” within each period; there is discounting only across periods.

Let the fixed cost of entry be $F = \bar{F} + \varepsilon$, where $\bar{F} > 0$ and ε is iid with $E(\varepsilon) = 0$ and CDF $\rho(\cdot)$ with support $[-\bar{F}, \infty]$. Each entrant knows her own fixed cost of entry *before* entering. An entrant’s fixed cost is private information, but all entrants know the distribution of fixed costs. Thus, entrants are heterogenous. This property somewhat complicates the space of strategic interactions because, in addition to the game between incumbents and entrants, it allows for a non-trivial game among the entrants themselves. In particular, entrants are not symmetric, so, despite the sequential structure, they are unable to foresee the entry process with certainty within a period. In previous studies, the symmetry of entrants (before entering) coupled with the sequential structure often led to entry being a deterministic function of the state of the game in equilibrium. In this model, on the contrary, the n firms in the market in round l (that is, the incumbents and the entrants who have entered before round $l + 1$) will expect an entrant to enter in the next round with probability $\rho_{l+1,n}$, which is determined endogenously. In sum, heterogeneity before entry induces a non-degenerate distribution of the number of entrants in each state.⁹

Assumption 3A is, therefore, modified as follows:

⁹Ex ante heterogeneity is essential for entry to have a non-trivial impact in the game. If entrants were unaware of their own fixed costs before entering, they would have identical entry rules. In this simple model, this would result in a deterministic entry process and an absorbing steady state when N is sufficiently large. An alternative way to obtain a non-degenerate distribution of entrants in each state would be simultaneous entry where entrants use mixed strategies.

Assumption 3B. *In each period there are N potential entrants, which enter sequentially at the beginning of the period. The l -th entrant enters with probability $\rho_{l,n}$ when there are n firms in the market at the end of the previous (production or entry) round.*

The summary of timing within a period is as follows: 1. Failures and exits occur. 2. If there were n firms in the last period and k failed, then the first entrant enters with probability $\rho_{1,n-k}$. Observing the number of firms in the last entry round, the l th entrant enters with probability $\rho_{l,n}$ if there are n firms in the $l-1$ th entry round. 3. After the final entry round, the production round is played among the n active firms (entrants plus incumbents).

Assumption 1, 2B, 3B are maintained throughout this section. As before, the strategies are assumed to be Markov and I focus on symmetric Markov Perfect Equilibrium (MPE). Let the equilibrium value function at the end of the l th entry round be $W_l(n)$. (That is, n includes the l th entrant if it has entered the market.) Then,

$$W_l(n) = W_{l+1}(n+1)\rho_{l+1,n} + W_{l+1}(n)(1 - \rho_{l+1,n}). \quad (4)$$

The sequential nature of entry facilitates a simple recursive formulation of the entry process. In particular, for the first entrant the value of being in the market is just the expected equilibrium firm value over the distribution of the second entrant's entry decision, which in turn depends on the third entrant's decision and so on until the last entry round. As there is no further entry after the last entry round of a period, $W_N(n) = V^e(n)$, where $V^e(n)$ is the (symmetric) equilibrium value function of the firm in the production round and defined as

$$V^e(n) = \pi(n) - g(x^e(n)) + \beta x^e(n) \sum_{k=0}^{n-1} [W_1(n-k+1)\rho_{1,n-k} + W_1(n-k)(1 - \rho_{1,n-k})] \binom{n-1}{k} (1 - x^e(n))^k (x^e(n))^{n-1-k} \quad (5)$$

where $x^e(n)$ is the optimal level of investment. The e superscript stands for entry. Equation (5) is similar to (1), except the continuation payoff is now the expected equilibrium firm value over

the distributions of the number of failing firms at the end of the period and the first entrant's entry decision. First, I establish the strong monotonicity of the value functions.

Proposition 5. *$V^e(n)$ and $W_l(n)$ are strictly decreasing in n . Furthermore, $W_l(n)$ is strictly increasing in l .*

Proof. See Appendix B. □

I can now fully characterise the entry process. The l th entrant enters if $W_l(n+1) - F > 0$, that is, it enters with probability $\rho_{l,n} = \rho(W_l(n+1) - \bar{F})$ when there are n firms in the market at the end of the previous (production or entry) round. It is not difficult to see that $0 < W_l(n) < \infty$, and therefore $0 < \rho_{l,n} < 1$. Consequently, entry is a (stochastic) threat in all states. From Proposition 5 it follows that $\rho_{l,n} < \rho_{l+1,n}$ and $\rho_{l,n} > \rho_{l,n+1}$. In other words, within a period, *ceteris paribus*, entrants enter with greater probability if closer to the production round and when the market is more concentrated. Both results are intuitive. An entrant closer to the production round enters with higher probability because the uncertainty regarding the remaining potential entries in that period is smaller. Also, more concentrated markets naturally attract more entry as the value functions are decreasing in n . These may seem to be countervailing effects. However, from (4) and Proposition 5 it is immediate that $\rho_{l,n} > \rho_{l+1,n+1}$. Therefore, if entry occurs in a given entry round, it will unambiguously lower the probability of further entry.

Now it is straightforward to prove the main result of the article, the strong monotonicity of the equilibrium investment schedule, the analogue of Proposition 3 in the presence of stochastic entry.

Proposition 6. *Firms' investment level is a strictly increasing function of market concentration, that is $x^e(n+1) < x^e(n)$.*

Proof. The proof is very similar to that of Proposition 3. □

Therefore, entry that is stochastically intensifying with concentration does not break the strong monotonicity of the equilibrium investment profile. This is somewhat surprising because, although there are potential rewards for survival, market concentration now frustrates survivors in three fundamental ways: it can bring less profit, it generates rivals that are harder to outlive, and also attracts more entry. Yet, in equilibrium these effects are dominated by the fact that there are fewer rivals to outlast. The model describes very intuitive industry dynamics. In fragmented markets, the equilibrium value of the firm is low and thus the intensity of entry is low too, while firms invest little. The market experiences a shake-out. However, as the industry becomes concentrated, firms invest more, fail with smaller probability and entry also intensifies. This prevents the market from becoming too concentrated for a prolonged time. Consequently, the model exhibits continuous turbulence in the form of failures and entry even in the limit. Now I turn to the analysis of the long run behaviour of the model.

As noted before, the *ex ante* heterogeneity of entrants introduces a non-degenerate distribution of entrants in each state. This complicates the stochastic structure of the model. In order to facilitate an analytically tractable analysis of the steady state dynamics, I apply continuous time approximation. In particular, I use the equilibrium properties of the discrete time game above to derive transition probabilities when each period of the game is infinitely small. In essence, I am going to transform the discrete time Markov transition kernel into a continuous time analogue and will prove that the game produces a birth and death process.

Let's introduce explicitly the length of a period dt . Note that the results above were obtained in the special case when $dt = 1$. I am going to relate the dynamics to birth and death processes, so it is convenient to redefine the probability of failure as $1 - x^e(n) = \theta_n$. Therefore, in a period a firm will fail with probability $\theta_n dt$ and will survive with probability $1 - \theta_n dt$. Also, the l th entrant enters with probability $\rho_{l,n} dt$ when there are n firms in the market. In the next proposition, I establish the existence of a unique steady state distribution.

Proposition 7. *There exists a unique stationary steady state distribution P , with probability mass function $P_n = \frac{\lambda_{n-1}}{\mu_n} P_{n-1}$ for $n \geq 1$ with $P_0 = \left(1 + \sum_{n=1}^{\infty} \prod_{i=0}^{n-1} \frac{\lambda_i}{\mu_{i+1}}\right)^{-1}$, and $E(n) < \infty$, where $\mu_n = n\theta_n$ and $\lambda_n = \sum_l \rho_{l,n}$.*

Proof. See Appendix B. □

The parameters μ_n and λ_n are the intensity parameters of the Markov process: μ_n can be interpreted as the average number of failures (“deaths”), and λ_n as the average number of entries (“births”) in a period with n firms. Although the entry process is sequential (a series of Bernoulli trials), it can be thought of as a binomial process with parameters N and $\bar{\rho}_n$ where $\bar{\rho}_n = \sum_l \rho_{l,n}/N$. Then it is easy to see that the process is essentially a well known death-birth process. We can think of $\mu_n P_n$ as the flow of exits from the state with n firms, and $\lambda_{n-1} P_{n-1}$ as the flow of new entries when there are $n-1$ firms. In the steady state distribution these two flows are equal. Equivalently, the likelihood ratio P_n/P_{n-1} is equal to the ratio of new entries from state $n-1$ over the exits from state n .

Proposition 8. *The most frequently visited state in steady state, that is, the mode of the steady state distribution is $n^* = \max\{n : \lambda_{n-1} > \mu_n\}$.*

Proof. By Proposition 6 μ_n is strictly increasing, and by Proposition 5 λ_n is strictly decreasing in n . Furthermore, $\lim_{n \rightarrow \infty} \lambda_n = 0$ and $\lim_{n \rightarrow \infty} \mu_n = \infty$. As $P_n/P_{n-1} = \lambda_{n-1}/\mu_n$, the probability mass function is increasing while $\lambda_{n-1} \geq \mu_n$ and decreasing afterwards. As a result, the probability mass function reaches its maximum for the highest n for which $\lambda_{n-1} > \mu_n$. □

The average number of exits from a state is an increasing function of the number of firms, whereas the average number of entries is a decreasing function of the number of firms. This implies that the steady state likelihood ratio P_n/P_{n-1} decreases with the number of firms n . It follows that the most likely state is the state with the highest number of firms n for which

$P_n/P_{n-1} > 1$. This makes intuitive sense, because such a state n^* is the only state with the following property: the (average) number of firms entering from n^*-1 is larger than the number of firm exiting from n^* , whereas the number of firms entering from n^* is smaller than the number of firms exiting from n^*+1 .

Some properties of the distribution can be readily deduced. As a consequence of the strong monotonicity of the entry and failure parameters, the shape of the distribution can be of two types. If no n exists such that $\lambda_{n-1} > \mu_n$, then the probability mass function is strictly decreasing. In this case, the mode of the distribution, that is, the most frequently visited state, is state zero (no firm in the market). If, on the contrary, $\lambda_{n-1} > \mu_n$ for some n , then the probability mass function is strictly increasing for small n , and strictly decreasing afterwards. These two cases suggest the pattern of a lognormal distribution: a high variance case will yield a strictly decreasing, whereas a lower variance an increasing then asymptotically decreasing curve.

4 Discussion

The model employs a stylised notion of competition to capture in a simple framework the dynamic interplay between firms' survival strategies and market structure. Perhaps surprisingly, the dynamic trade-off between current and future profits is sufficient in and of itself to explain some standard features of stochastic market evolution. Although the symmetric framework prohibits the model from addressing many interesting empirical phenomena of industry dynamics (e.g. variance of firm growth rates, size distribution, etc.), it is still able to rationalise two well-known empirical regularities. In particular, many industries are characterised by shake-outs and correlated entry and exit rates. So far the theoretical literature has explained shake-outs by the introduction of some innovation shocks and correlated entry and exit rates by fluctuations in demand/relative efficiency and displacement of existing technologies/products (Sutton 2007). In the current model, however, shake-outs and correlated entry and exit rates are not

driven by exogenous shocks, but are solely the results of firm behaviour.¹⁰

By assumption, the distribution of future states, conditional on the current state and investments, is independent of the prices (or quantities) that firms set in the market game. Similarly to many other studies (e.g. Ericson and Pakes 1995), this allows me to model the market game in a static framework and simply feed $\pi(n)$ into the dynamic optimization problem as a primitive of the stochastic dynamic game.¹¹ Beyond symmetry, Assumption 1 imposes very mild conditions on the admissible market games. Appendix A offers a simple example a la Salop, but it is important to emphasise that most other standard differentiated product settings, such as discrete choice¹² or CES demand specifications, naturally conform to Assumption 1.¹³

The present framework differs in a number of respects from the models of industry dynamics spearheaded by Ericson and Pakes (1995). There are two crucial features, however, that facilitate the particularly simple analytical structure of the current design. First, there is no firm heterogeneity in the model. Second, the firm's own state is a binary variable (success or failure).¹⁴ These two features simplify the state space considerably and allow me to capture all payoff relevant information in a single variable, the number of firms. This leads to a particularly simple analytical structure, which, in contrast to the Ericson and Pakes setup, delivers definite comparative-static predictions. It is straightforward to test these predictions in a reduced form framework (e.g. a cross section of industries in the spirit of Sutton 1998). However, it is also possible to introduce heterogeneity and estimate the model structurally, while choosing from a rich set of empirical specifications for the stage game.¹⁵

¹⁰Correlated entry and exit rates follow from the observation that failure waves are followed by intense entry and *vice versa*. Also, from Proposition 8, the most frequently visited state, the mode of the steady state distribution, is the market structure where entry and failure rates are approximately equal.

¹¹This is a simplification that rules out the analysis of markets with e.g. learning by doing and network effects.

¹²A simple example can be found in Doraszelski and Pakes (2007) and in Appendix A.

¹³Chen and Riordan (2008) provide sufficient conditions in discrete choice and Chen and Riordan (2007) in spatial settings.

¹⁴This is not as radical an assumption as it may seem: in the presence of entry failure need not be interpreted literally. See footnote 5.

¹⁵Typically, both observed and unobserved heterogeneity are essential in a structural framework. Note that, however, observed heterogeneity needs not complicate the state space of the model. For instance, in Aguirre-gabiria and Mira (2007) there is no firm heterogeneity, the variation necessary to identify structural parameters comes from variation across (independent) markets. Therefore, like in my model, a firm in a local market can base its decision solely on the number of firms, facilitating a particularly simple dynamic optimisation problem.

The basic features of the model can potentially suit many applications. As mentioned before, the framework is a natural description of the financial industry where firms, choosing their risk profile, trade off higher profits for less risk. Moreover, the model is directly applicable in markets where severe asymmetric information problems lend a crucial role to reputation and the disciplinary mechanism of firm failure. It is also straightforward to interpret the investment in survival as R&D in some markets. In particular, the present model would naturally suit R&D applications where failure to innovate in a period results in either literal firm failure or the firm having to incur some “entry cost” (e.g. buying patents) if it wishes to compete in the market again. This is most likely to hold in markets where the fast paced technological development requires ongoing success in innovation to maintain market presence (e.g. IT, smart phones).¹⁶ If a period of the game spans a longer horizon, then this feature is plausible in most industries where only successful innovators can ensure firm survival in the long run. Similarly, one can envisage a game for access to the market where continuous investment is necessary to maintain access. The investment cost can, for instance, represent advertising, where advertising increases the probability that the firm will be remembered next period. If a firm gets forgotten, it is practically out of the market and has to sink an entry fee if it wishes to enter the game for access again.¹⁷

Due to its simple structure, the model is particularly well suited for further analytical work. For instance, in banking the strategic aspect of systemic risk build-up is unexplored. In particular, it is often overlooked that a bank’s risk taking incentives can be affected by rivals’ risk taking in a strategic space that is defined by the dynamics of market structure. The current model provides some useful insights. But how do the incentives to take risk change when system failure is not allowed? In other words, how much risk does a bank take if it knows that in case of failure it will be in fact bailed out if its rivals fail too? Preliminary analysis shows that in the presence of systemic bail-out, in more concentrated markets the game of strategic substitutes

¹⁶This notion of R&D competition is quite different from a standard (R&D, patent) race, where typically there is a technological advantage, being “ahead” or “behind” of rivals, e.g. Harris and Vickers (1987), Hörner (2004), Konrad and Kovenock (2009).

¹⁷This is similar to the notion of “awareness advertising” in Doraszelski and Markovich (2007).

can turn into a game of strategic complements inducing more risk taking. I will explore these questions in future work.

5 Banking industry

The present model is highly stylised in order to capture features across several industries and hence it is not designed to give a detailed description of any particular market. However, in what follows I discuss some empirical results from the banking industry which are consistent with the model's predictions. The financial industry in general is a particularly good example of the present model because financial firms routinely trade off higher profits against less risk. Indeed, firm failure, as a direct consequence of risk exposure, has been one of the major driving forces of industry dynamics in banking.

In the 1980s, the world-wide deregulation process in effect reintroduced competition in the industry by lifting geographical restrictions and also controls on interest rates and investment activities. In the USA, for instance, liberalization led to a market shake-out and subsequently further exits (i.e. failures, but also mergers and acquisitions): the number of banks decreased by 48% between 1984 and 2003, despite the more than 3000 de novo entries.¹⁸ (Jones and Critchfield 2008) The empirical literature often uses concentration as a proxy for (lack of) competition and most of these studies find that concentration is associated with less risky banking. (e.g. Rhoades and Rutz 1982, Bergstresser 2001, Maudos and Guevara 2004, Brewer and Jackson 2006, Beck et al 2005, 2006, Braggion et al 2011)¹⁹ However, very few articles have attempted to separate the effects of competition from that of concentration.²⁰ Claessens and Laeven (2004) was the first to observe that competition does not necessarily lessen with concentration in banking: in a panel of 50 countries over the period of 1994-2001 they find a

¹⁸Interestingly, traditional arguments such as scale economies do not seem to explain the steady process of market concentration of the last three decades. (e.g. Bikker 2004 and Amel et al 2004)

¹⁹Braggion et al (2011) shows that banks in more concentrated markets had less risky portfolios in the UK between 1885-1925. This is interesting because there was no bank regulation, supervision and deposit insurance in this period.

²⁰Note that unlike my model, most of the theoretical banking literature does not make distinction between market structure and competition. (see the survey Beck 2008)

positive relationship between competition and market concentration. Their results suggested that concentration is a poor proxy for competition and in fact competition and concentration can have independent effects on risk taking. Indeed, building on this finding, Schaek et al (2009) show in a panel of 45 countries over the period 1980-2005 that concentration reduces risk in banking when controlling for competition²¹ (along with a host of institutional and macroeconomic characteristics). These findings emerge endogenously in my model as higher concentration is associated with less risk taking (higher x) and a more stable market (i.e. less failure), while competition can also increase (i.e. net profits $\pi(n) - g(x_i)$ decrease) with concentration. Furthermore, the current model provides an argument for why banking markets have become so concentrated, without relying on traditional arguments such as scale economies, reduced competition, entry barriers, 'too big to fail' effects, etc.

Banking is not the only example. For instance, other markets with severe moral hazard problems, such as the audit market or the market of Health Maintenance Organizations, appear to produce some empirical findings consistent with the model's predictions (audit: e.g. GAO 2008; HMO: e.g. Jin 2005, Dafny et al 2009). Furthermore, although the current framework can only be taken as an abstract model of innovation, it predicts a robust positive relationship between innovation activity and market concentration, an empirical regularity observed in most markets (see the survey by Sutton 2007). As such, it provides a theoretical underpinning for the much tested Schumpeterian Hypothesis, according to which market concentration increases innovation effort. In the current model, however, the incentives to innovate do not operate through the typical channels of previous studies, such as scale economies, firm size or market power.

6 Conclusion

In this study, market rivalry is modelled as a tournament. Firms' evolving expectations of future market structure drive their investments in survival, which, in turn, fuels market structure

²¹Both Claessens and Laeven (2004) and Schaek et al (2009) measure competition by the Panzar and Rosse H-statistic. The H statistic estimates the extent to which changes in input prices are reflected in revenues.

dynamics. When too many rivals upset the prospects of the contest, firms gamble on their survival while the intensity of entry is low. This leads to shake-outs in fragmented markets. Failing rivals, however, raise the stakes for survivors; hence, the more concentrated the market becomes, the more each firm strives to survive. This results in an investment schedule which is increasing with concentration in equilibrium. The resulting dynamics keeps the market in ongoing turbulence, where market concentration is associated with more intense rivalry: concentrated markets generate less profit, more formidable rivals, and also more entry.

The model provides an alternative explanation for why markets may become concentrated over time. Rather than scale economies and entry barriers, strategic gambling may explain the convergence towards concentration that we observe, for instance, in some markets with moral hazard, such as banking, audit and health care. In the present model, the endogenous nature of the tournament allows one to investigate explicitly the (steady state) probability distribution of subcontests (i.e. market structures). Although prize(s) are won in finite time (almost surely), the prospect of winning the prize can intensify the contest for survival such that the prize may not be won for a very long time. This means that the contest can produce dynamics in practice that are observationally equivalent to competition increasing with market concentration.

Appendix A Example: unobserved quality and reputation

The aim of this appendix is to provide an example of the stage game in the baseline model. The model below is a straightforward variant of unobserved quality models such as e.g. Allen (1984), Hörner (2002) and Rob and Fishman (2005).

The model

As in Salop (1979), there is a continuum of consumers located uniformly along a circle, their measure is normalised to one. Each consumer wishes to buy one product each period, the quality of which can be high $u > 0$ or low 0. Neither consumers nor firms observe quality before consumption. Products are horizontally differentiated along some observable characteristics,

that are modelled as travel distances. Denote the distance between a consumer and firm i by $s_i \in (0, 1)$. The *ex ante* net benefit to consume the good produced by firm i at time t is $U_i^t = q_i^t u - p_i^t - cs_i^t$, where q_i^t is the consumer's belief of buying a high quality product from firm i , $p_i^t \in \mathbb{R}_+$ is the price, and $c \geq 0$ measures the travel cost.²² Consumers' outside option is normalised to zero so they only buy if their expected utility is non-negative. This utility function implies identical consumer attitudes towards (unobserved) quality, because every consumer obtains negative utility from consuming a low quality product at a positive price, regardless of her distance from firms.

At time t there are n^t firms symmetrically located on the circle.²³ Let $N^t = \{1, \dots, n^t\}$ be the set of firms in period t . The cost of production is zero. In each period, firm i can invest x_i^t in quality at a cost of $g(x_i^t)$; x_i^t is private information (moral hazard). After investing x_i^t firm i produces low quality with probability $1 - x_i^t$. The per-period profit net of investment for firm i is $\hat{\pi}(p_i^t; p_{-i}^t, \gamma^t)$, where $p_{-i}^t = [p_j^t]_{j \neq i}$ and γ^t is a vector of consumers' actions as defined below.

Timing: In each period, 1. Firms decide to quit or stay. Incumbents relocate symmetrically. 2. Firms choose prices and x simultaneously. 3. Consumers choose firms. 4. Consumption takes place and consumers and firms observe current period quality.

Markov Strategies: I will look at Markov strategies that only depend on last period qualities. Define the set of (last period) quality histories as $H^t = \times_{i \in N^{t-1}} \{u^{t-1}, 0\}$. A consumer chooses which firm to buy from as a function of the history of qualities, her distance from firms, and current prices. Her strategy is described by the mapping $\gamma : H^t \times [\times_{i \in N^t} (0, 1)] \times [\times_{i \in N^t} \mathbb{R}_+] \rightarrow N^t$. Having observed last period qualities, firms decide to quit or stay. Then incumbents choose prices and set the level of investment in quality. Thus, the strategy of firm i consists of three mappings: $\tau_i : H^t \rightarrow \{Quit, Stay\}$, $p_i : H^t \times N^t \rightarrow \mathbb{R}_+$ and $x_i : H^t \times N^t \rightarrow [0, 1]$.

²² c is an index of the degree of product differentiation. If $c = 0$, then goods are homogeneous, whereas if $c > 0$ products are differentiated.

²³I conjecture it is possible to extend the model and allow firms to choose their location. Economides (1989) shows that with free location choice, there exists a symmetric equilibrium in locations and prices.

Symmetric (Subgame Perfect) Nash Equilibrium in Markov strategies

Consumers: Do not buy from a firm which has produced bad quality. (More generally, do not buy from firm i if $U_i^t < 0$.) Buy from a firm which maximises expected utility; that is, choose firm i such that $U_i^t \geq U_{j \neq i}^t, \forall i, j \in N^t$ and $U_i^t \geq 0$. Moreover, consumers' beliefs are such that 1) if a firm has produced bad quality, it will always produce bad quality with probability one (i.e. $q_i^t, q_i^{t+1}, \dots = 0$), 2) in period t , firms produce good quality with probability $q_i^t = q^t = x_*^t$, if they haven't produced bad quality before.

Firms: Quit if you produced bad quality last period. If you produced good quality last period, choose p^t such that $p^t = r(p_{-i}^t, \gamma^t) \in \arg \max_{p_i^t} \hat{\pi}(p_i^t; p_{-i}^t, \gamma^t)$. If you produced good quality last period, choose x^t which maximises the dynamic programme of (1), where $\pi(n) \equiv \hat{\pi}(r(p_*^t, \gamma_*^t), p_*^t, \gamma_*^t)$.^{24,25}

Appendix B Proofs

The following Lemmas will be useful in what follows.

Lemma 1. $xg'(x) > g(x)$ for $x > 0$.

Proof. Recall that $g''(\cdot) > 0$ by Assumption 2. Hence, the following holds for any two distinct points v, x in the domain: $g(v) > g(x) + g'(x)(v - x)$. Letting $v = 0$ and recalling that $g(0) = 0$ gives the inequality result. \square

²⁴A logit demand specification is equally simple. Suppose, as in Doraszelski and Pakes (2007), $U_{il} = U(y_l, z_i, p_i, q_i) = \kappa(q_i) - p_i - \epsilon_{il}$, where y_l is a consumer attribute (e.g. consumer's location), z_i is a product characteristic (e.g. a firm's location), $\kappa(q_i)$ is an increasing and bounded function representing the "mean utility" of firm i 's product (given identical consumer beliefs about unobserved quality of firm i) and $\epsilon_{il} = \iota(y_l, z_i)$ is the taste parameter. The demand for product i is then simply the sum of consumers who prefer product i to j : $d_i = \Pr \{y_l | U(y_l, z_i, p_i, q_i) \geq U(y_l, z_j, p_j, q_j), \forall j \neq i\}$. (Note that this is just the Salop setting above when $\kappa(q_i) = q_i u$ and $\epsilon_{il} = c \|y_l - z_i\| \equiv cs_i$.) If we assume that the taste parameters are independently and identically extreme value distributed across consumers and products, this yields the standard logit demand system. Following Doraszelski and Pakes (2007), equilibrium profits are then $\pi(q_i, q_{-i})$, which in turn is just $\pi(n)$ if the equilibrium is in fact symmetric.

²⁵Observe an important feature of this equilibrium: there is no strategic link between price and investment in quality. Given consumers' strategies firms don't use price as a signal of their investment, and given this, consumers rightly ignore prices when they form their beliefs about quality. This feature is similar to e.g. Rob and Fishman (2005).

Lemma 2. *In a symmetric equilibrium without entry, the value function is strictly decreasing: $V(n) < V(n-1)$.*

Proof. The proof is by induction. First, I show that $V(2) < V(1)$. Then supposing $V(n) < \dots < V(1)$, I'll show that $V(n+1) < V(n)$, which proves the claim.

Step 1. To prove $V(2) < V(1)$ observe that

$$\begin{aligned} V(2) - V(1) &= \pi(2) - g(x(2)) + \beta x(2)[V(2)x(2) + V(1)(1 - x(2))] - V(1) \\ &= \frac{\pi(2) - g(x(2)) + \beta x(2)V(1) - V(1)}{1 - \beta x(2)^2} < \frac{\pi(1) - g(x(2)) + \beta x(2)V(1) - V(1)}{1 - \beta x(2)^2} \leq 0 \end{aligned}$$

where the first inequality follows from Assumption 1 and the second from the fact that $\pi(1) - g(x(2)) + \beta x(2)V(1) \leq V(1)$ because $x(2)$ is the maximiser of the duopoly's dynamic programme, rather than the monopoly's.

Step 2. Next, suppose $V(n) < \dots < V(1)$. Then, I'll show that $V(n+1) < V(n)$. Let $x \equiv x(n+1)$ be the value which maximises the firm's dynamic program in a symmetric equilibrium when there are $n+1$ firms in the market. Then,

$$V(n+1) = \pi(n+1) - g(x) + \beta \sum_{k=0}^n V(n+1-k) \binom{n}{k} (1-x)^k x^{n+1-k}$$

Therefore,

$$\begin{aligned} V(n+1) &\leq \pi(n) - g(x) + \beta \sum_{k=0}^{n-1} V(n-k) \binom{n-1}{k} (1-x)^k x^{n-k} \\ &\quad + \beta \sum_{k=0}^n V(n+1-k) \binom{n}{k} (1-x)^k x^{n+1-k} - \beta \sum_{k=0}^{n-1} V(n-k) \binom{n-1}{k} (1-x)^k x^{n-k} \end{aligned} \tag{B.1}$$

where the inequality follows from Assumption 1 again. Now, observe that the expression on the right hand side in the first line of (B.1) is less than $V(n)$ because x maximises the value function when there are $n+1$ firms, rather than n . Thus, from (B.1) we get

$$V(n+1) - V(n) \leq \beta \sum_{i=-1}^{n-1} V(n-i) \binom{n}{i+1} (1-x)^{i+1} x^{n-i} - \beta \sum_{i=0}^{n-1} V(n-i) \binom{n-1}{i} (1-x)^i x^{n-i}$$

where I switched indices setting $k = i + 1$ in the first term and $k = i$ in the second. Next, using Pascal's identity

$$\binom{n}{i+1} = \binom{n-1}{i+1} + \binom{n-1}{i} \quad (\text{B.2})$$

and the convention $\binom{n}{l} = 0$ for $l < 0$ and $l > n$, the inequality can be rearranged as

$$V(n+1) - V(n) \leq \beta \sum_{i=-1}^{n-1} V(n-i) \binom{n-1}{i+1} (1-x)^{i+1} x^{n-i} - \beta \sum_{i=0}^{n-1} V(n-i) \binom{n-1}{i} (1-x)^i x^{n+1-i}$$

The last element in the first sum can be dropped because it's zero. Then, switching the index back again, set $i = k - 1$ in the first sum and $i = k$ in the second. This yields:

$$\begin{aligned} V(n+1) - V(n) &\leq \beta \sum_{k=0}^{n-1} [V(n+1-k) - V(n-k)] \binom{n-1}{k} (1-x)^k x^{n+1-k} \\ &= \frac{\beta \sum_{k=1}^{n-1} [V(n+1-k) - V(n-k)] \binom{n-1}{k} (1-x)^k x^{n+1-k}}{1 - \beta x^{n+1}} < 0 \end{aligned}$$

where the last inequality holds by the induction hypothesis. \square

Proof of Proposition 1

By inspection of (1), it is immediate that Blackwell's sufficiency conditions, namely monotonicity and discounting, hold. Therefore, the value function exists and unique. Also, the second order condition is simply $-g''(x) < 0$ (Assumption 2). This implies that the reaction functions are unique. It remains to show that these reaction functions define a symmetric equilibrium of the game in pure strategies. This follows from the symmetric reaction functions being continuous and downward sloping. Let's write the (interior) first order condition to the programme (2) in the following way

$$f_i(x_i; x_j, x_l, n) \equiv \beta x_j \sum_{k=0}^{n-2} V(n-k) c_k^{(n-2)} + \beta (1-x_j) \sum_{k=0}^{n-2} V(n-1-k) c_k^{(n-2)} - g'(x_i) = 0$$

where $c_k^{(n-2)} = \Pr(k|x_l)$ is the probability mass function of the convolution of $n-2$ Bernoulli

trials ($\sum_{k=0}^{n-2} c_k^{(n-2)} = 1$) and $x_l = [x_h]_{h \neq i, j}$. Continuity is obvious. Furthermore, the reaction functions are downward sloping because, by the Implicit Function Theorem,

$$\frac{\partial x_i}{\partial x_j} = -\frac{\partial f_i(x_i; x_j, x_l, n)}{\partial x_j} / \frac{\partial f_i(x_i; x_j, x_l, n)}{\partial x_i} = \frac{\beta \sum_{k=0}^{n-2} [V(n-k) - V(n-1-k)] c_k^{(n-2)}}{g''(x_i)} < 0$$

where the inequality follows from Lemma 2. Note that the reactions functions are linear ($\partial x_i / \partial^2 x_j = 0$), hence the symmetric equilibrium is unique if $|\partial x_i / \partial x_j| \neq 1$.

Proof of Proposition 3

The proof consists of two parts. In the first part I show that the left hand side of the first order condition (2) decreases with x . In the second, I prove that it also decreases stochastically with n . As a result, higher n must imply lower x in equilibrium. Let $f(x; n, \beta)$ denote the left hand side of the FOC in (2) with $\zeta = \varrho = 0$:

$$f(x; n, \beta) \equiv \beta \sum_{k=0}^{n-1} V(n-k) \binom{n-1}{k} (1-x)^k x^{n-k-1} - g'(x) \quad (\text{B.3})$$

Step 1. First, I show that $\partial f(x; n, \beta) / \partial x < 0$. To see this, differentiate (B.3)

$$\frac{\partial f(x; n, \beta)}{\partial x} = \beta \frac{\partial}{\partial x} \sum_{k=0}^{n-1} V(n-k) \binom{n-1}{k} (1-x)^k x^{n-1-k} - g''(x)$$

The last term is negative by Assumption 2. The first term is proportional to

$$\begin{aligned} \frac{\partial}{\partial x} \sum_{k=0}^{n-1} V(n-k) c_k^{(n-1)} &= \sum_{k=0}^{n-1} V(n-k) \binom{n-1}{k} (n-1-k) (1-x)^k x^{n-2-k} \\ &\quad - \sum_{k=0}^{n-1} V(n-k) \binom{n-1}{k} k (1-x)^{k-1} x^{n-1-k} \end{aligned}$$

where $c_k^{(n-1)} = \binom{n-1}{k} (1-x)^k x^{n-1-k}$. Dropping the $(n-1)$ th element in the first sum and

the first element in the second sum, which are both zero, we obtain

$$\begin{aligned} \frac{\partial}{\partial x} \sum_{k=0}^{n-1} V(n-k) c_k^{(n-1)} &= (n-1) \sum_{k=0}^{n-2} V(n-k) \frac{(n-2)!}{k!(n-2-k)!} (1-x)^k x^{n-2-k} \\ &\quad - (n-1) \sum_{k=1}^{n-1} V(n-k) \frac{(n-2)!}{(k-1)!(n-1-k)!} (1-x)^{k-1} x^{n-1-k} \end{aligned}$$

Substitute $i = k$ in the first sum and $i = k - 1$ in the second sum. Then

$$\frac{\partial}{\partial x} \sum_{k=0}^{n-1} V(n-k) c_k^{(n-1)} = (n-1) \sum_{i=0}^{n-2} [V(n-i) - V(n-1-i)] \binom{n-2}{i} (1-x)^i x^{n-2-i}$$

The right hand side is negative by Lemma 2. Therefore $\partial f(x; n, \beta) / \partial x < 0$.

Step 2. Next, I'll show that $f(\cdot)$ is (stochastically) decreasing in n . In particular, for any $x \in (0, 1)$, $f(x; n+1, \beta) < f(x; n, \beta)$. For this, we need

$$\begin{aligned} \sum_{k=0}^n V(n+1-k) \binom{n}{k} (1-x)^k x^{n-k} &< \sum_{i=0}^{n-1} V(n-i) \binom{n-1}{i} (1-x)^i x^{n-1-i} \\ &= \sum_{k=1}^n V(n+1-k) \binom{n-1}{k-1} (1-x)^{k-1} x^{n-k} \end{aligned} \quad (\text{B.4})$$

where the equality follows from setting $i = k - 1$. In other words, the expected value of $V(n+1) < \dots < V(1)$ with CDF $B(v; n, x)$ is smaller than the expected value of $V(n) < \dots < V(1)$ with CDF $B(v; n-1, x)$, where:

$$\begin{aligned} B(v; n, x) &= \sum_{k=0}^v b(k; n, x) = \sum_{k=0}^v \binom{n}{k} (1-x)^k x^{n-k} \\ B(v; n-1, x) &= \sum_{k=1}^v b(k; n-1, x) = \sum_{k=1}^v \binom{n-1}{k-1} (1-x)^{k-1} x^{n-k} \end{aligned}$$

Now, because $V(\cdot)$ is strictly increasing in k (Lemma 2), (B.4) holds if $B(v; n-1, x)$ with support $v = 1, \dots, n$ (first order) stochastically dominates $B(v; n, x)$ with support $v = 0, \dots, n$; that is, $B(v; n, x) \geq B(v; n-1, x)$ for $v = 0, \dots, n$ and with strict inequality for some v . This inequality is shown to hold by induction. It can be easily seen that $B(0; n, x) - B(0; n-1, x) = x^n$

and $B(1; n, x) - B(1; n - 1, x) = (n - 1)(1 - x)x^{n-1}$. Therefore, the inductual hypothesis is

$$B(v; n, x) - B(v; n - 1, x) = \binom{n-1}{v} (1-x)^v x^{n-v} \quad (\text{B.5})$$

As $B(v+1; n, x) = B(v; n, x) + b(v+1; n, x)$, therefore,

$$B(v+1; n, x) - B(v+1; n-1, x) =$$

$$\begin{aligned} &= \binom{n-1}{v} (1-x)^v x^{n-v} + \binom{n}{v+1} (1-x)^{v+1} x^{n-v-1} - \binom{n-1}{v} (1-x)^v x^{n-v-1} \\ &= \left[\binom{n}{v+1} - \binom{n-1}{v} \right] (1-x)^{v+1} x^{n-v-1} = \binom{n-1}{v+1} (1-x)^{v+1} x^{n-v-1} \geq 0 \end{aligned}$$

where after the first equality sign I used the inductual hypothesis (B.5), and after the last equality sign I used Pascal's identity (B.2). The inequality above is strict for all $v < n - 1$, so inequality (B.4) holds, and we can conclude that $f(x; n, \beta)$ is decreasing in both x and n .

Proof of Proposition 4

From the first order condition (B.3),

$$\begin{aligned} \frac{\partial f(x; n, \beta)}{\partial \pi(n)} &= \beta x^{n-1} \frac{\partial V(n)}{\partial \pi(n)} = \frac{\beta x^{n-1}}{1 - \beta x^n} > 0, \\ \frac{\partial f(x; n, \beta)}{\partial \pi(1)} &= \beta \sum_{k=0}^{n-1} \frac{\partial V(n-k)}{\partial \pi(1)} \binom{n-1}{k} (1-x)^k x^{n-k-1} > 0 \end{aligned}$$

The second inequality follows from a simple induction argument as follows. Clearly, $\partial V(1)/\partial \pi(1) = 1/(1 - \beta x_1) > 0$. Now suppose $\partial V(1)/\partial \pi(1), \dots, \partial V(n-1)/\partial \pi(1) > 0$. Then,

$$\frac{\partial V(n)}{\partial \pi(1)} = \frac{\beta}{1 - \beta x^n} \sum_{k=1}^{n-1} \frac{\partial V(n-k)}{\partial \pi(1)} \binom{n-1}{k} (1-x)^k x^{n-k} > 0$$

where the inequality holds by the induction hypothesis. Hence, $\partial f(x; n, \beta)/\partial \pi(1) > 0$. A very similar argument shows that $\partial f(x; n, \beta)/\partial \beta > 0$. Then, using the Implicit Function

Theorem, $\partial f(x; n, \beta)/\partial x < 0$ (Proposition 3, Step 1) and the inequalities derived above, we have

$$\begin{aligned}\frac{dx(n)}{d\pi(n)} &= -\frac{\partial f(x; n, \beta)}{\partial \pi(n)} / \frac{\partial f(x; n, \beta)}{\partial x} > 0, & \frac{dx(n)}{d\pi(1)} &= -\frac{\partial f(x; n, \beta)}{\partial \pi(1)} / \frac{\partial f(x; n, \beta)}{\partial x} > 0, \\ \frac{dx(n)}{d\beta} &= -\frac{\partial f(x; n, \beta)}{\partial \beta} / \frac{\partial f(x; n, \beta)}{\partial x} > 0\end{aligned}$$

Proof of Proposition 5

First, I consider the finite version of the dynamic programme and by a straightforward inductual argument I establish that $V_t^e(n-1) - V_t^e(n) \geq 0$ and $W_{l,t}(n-1) - W_{l,t}(n) \geq 0$ for all t, l and n , where $V_t^e(\cdot)$ and $W_{l,t}(\cdot)$ are value functions when there are t periods remaining. Then, by standard dynamic programming arguments, $\lim_{t \rightarrow \infty} V_t^e(n) = V^e(n)$, so it is proven that $V^e(n-1) - V^e(n) \geq 0$. Similarly $W_l(n-1) - W_l(n) \geq 0$. Next, I show that $0 < V^e(n), W_l(n) < \infty$ and hence $0 < \rho_{l,n} < 1$ for all n, l . Lastly, the strict monotonicity of the value functions will be established. In what follows I drop the superscript and the argument of $x^e(n)$ in order to ease notation so $x \equiv x^e(n)$.

Step 1 By Assumption 1, $V_0^e(n-1) - V_0^e(n) \geq 0$ for all n because $V_0^e(n) = \pi(n)$. Now noting that $V_0^e(n) = W_{N,0}(n)$, from (4) a straightforward backwards induction shows that $W_{l,0}(n-1) - W_{l,0}(n) \geq 0$ for all l and n . So suppose $V_{t-1}^e(n-1) - V_{t-1}^e(n) \geq 0$ and $W_{l,t-1}(n-1) - W_{l,t-1}(n) \geq 0$ for all l and n . Then, using the same transformations as in Step 2, proof of Lemma 2, one can show that

$$\begin{aligned}V_t^e(n) - V_t^e(n-1) &\leq \\ &\beta \sum_{k=0}^{n-2} ([W_{1,t-1}(n-k+1)\rho_{1,n-k} + W_{1,t-1}(n-k)(1-\rho_{1,n-k})] \\ &\quad - [W_{1,t-1}(n-k)\rho_{1,n-k-1} + W_{1,t-1}(n-k-1)(1-\rho_{1,n-k-1})]) \binom{n-2}{k} (1-x)^k x^{n-k} \\ &\leq 0\end{aligned}\tag{B.6}$$

The last inequality follows from the induction hypothesis, $W_{1,t-1}(n-k+1) \leq W_{1,t-1}(n-k) \leq$

$W_{1,t-1}(n-k-1)$ for all $0 \leq k \leq n-2$. Noting that $V_t^e(n) = W_{N,t}(n)$, from (4) a straightforward application of backwards induction shows that $W_{l,t}(n-1) - W_{l,t}(n) \geq 0$. Therefore, for all t, l and n , $V_t^e(n-1) - V_t^e(n) \geq 0$ and $W_{l,t}(n-1) - W_{l,t}(n) \geq 0$. The mapping $V_{t+1}^e(n) = TV_t^e(n)$ is a contraction, so $\lim_{t \rightarrow \infty} V_t^e(n) = V^e(n)$ uniformly. Then $V^e(n-1) - V^e(n) \geq 0$ and similarly, $W_l(n-1) - W_l(n) \geq 0$ for all n and l . Weak monotonicity is established.

Step 2 Substituting the first order condition back into (5) yields $V^e(n) = \pi(n) - g(x) + xg'(x)$. From Lemma 1 and the fact that $x \neq 0$ in equilibrium, $V^e(n) > \pi(n) \geq 0$, where the second inequality follows from Assumption 1. As no firm can do better than collecting monopoly profits ad infinitum while investing $x = 0$, $V^e(n) \leq \pi(1)/(1 - \beta) < \infty$ for all n . But this just implies $0 < W_l(n) < \infty$ and hence $0 < \rho_{l,n} < 1$ for all n, l .

Step 3 In what follows, I show the strong monotonicity of $V^e(n)$ and $W_l(n)$. Inequality (B.6) holds at the limit too, so we can consider it without the time subscripts. Furthermore, from Assumption 1 for $n = 2$ the first inequality in (B.6) is actually strict (see Step 1, proof of Lemma 2), which can then be written as

$$V^e(2) - V^e(1) < \beta x^2 [W_1(3)\rho_{1,2} + W_1(2)(1 - \rho_{1,2})] - [W_1(2)\rho_{1,1} + W_1(1)(1 - \rho_{1,1})] \leq 0$$

The second inequality follows from what we have established earlier, that is $W_l(n-1) - W_l(n) \geq 0$ for all l . But then $W_{N-1}(1) = V^e(2)\rho_{N,1} + V^e(1)(1 - \rho_{N,1}) > W_{N-1}(2) = V^e(3)\rho_{N,2} + V^e(2)(1 - \rho_{N,2})$ because $W_N(n) = V^e(n)$ and $0 < \rho_{N,1}, \rho_{N,2} < 1$. Noting again $W_l(2) - W_l(3) \geq 0$ and $0 < \rho_{l,1}, \rho_{l,2} < 1$ for all l , this implies through backwards induction that $W_1(1) > W_1(2)$. As a consequence, there is at least one negative element on the right hand side of (B.6), which just means the second inequality in (B.6) is strict. Therefore, $V^e(n) < V^e(n-1)$ for all n . Following the logic of the argument above, it immediately follows that $W_l(n) < W_l(n-1)$ for all l and n .

Proof of Proposition 7

Denote the random variable of the number of failures as $\omega(t)$. Using the fact that $(1 - adt)^h = \sum_{i=0}^h \binom{h}{i} (-adt)^i = 1 - hadt + o(dt)$, where $\lim_{dt \rightarrow 0} o(dt)/dt = 0$, the transition probabilities in the production round are

$$\Pr(\omega(t + dt) = k | n) = \binom{n}{k} (\theta_n dt)^k (1 - \theta_n dt)^{n-k} = \begin{cases} 1 - n\theta_n dt + o(dt) & \text{if } k = 0 \\ n\theta_n dt + o(dt) & \text{if } k = 1 \\ o(dt) & \text{if } k > 1 \end{cases}$$

Similarly, letting the random variable $\psi(t)$ denote the number of entering firms, the transition probabilities in the entry process are

$$\Pr(\psi(t + dt) = j | n - k) =$$

$$= \begin{cases} \prod_{l=1}^N (1 - \rho_{l,n-k} dt) = 1 - \sum_l \rho_{l,n-k} dt + o(dt) & \text{if } j = 0 \\ \sum_{l=1}^N \rho_{l,n-k} dt \prod_{i < l} (1 - \rho_{i,n-k} dt) \prod_{i > l} (1 - \rho_{i,n-k+1} dt) = \sum_l \rho_{l,n-k} dt + o(dt) & \text{if } j = 1 \\ o(dt) & \text{if } j > 1 \end{cases}$$

Let the random variable $Z(t)$ denote the number of firms in the market at time t . Then $dZ(t)/dt = \psi(t) - \omega(t)$. The transition probabilities in a period when there are n firms in the market are as follows

$$\begin{aligned}
\Pr(Z(t+dt) = n+1 | Z(t) = n) &= [1 - n\theta_n dt + o(dt)] \left[\sum_l \rho_{l,n} dt + o(dt) \right] + o(dt) \\
&= \sum_l \rho_{l,n} dt + o(dt) \\
\Pr(Z(t+dt) = n-1 | Z(t) = n) &= [n\theta_n dt + o(dt)] \left[1 - \sum_l \rho_{l,n} dt + o(dt) \right] + o(dt) \\
&= n\theta_n dt + o(dt) \\
\Pr(Z(t+dt) = n | Z(t) = n) &= [1 - n\theta_n dt + o(dt)] \left[1 - \sum_l \rho_{l,n} dt + o(dt) \right] \\
&\quad + [n\theta_n dt + o(dt)] \left[\sum_l \rho_{l,n} dt + o(dt) \right] + o(dt) \\
&= 1 - n\theta_n dt - \sum_l \rho_{l,n} dt + o(dt) \tag{B.7} \\
\Pr(Z(t+dt) = n+m | Z(t) = n) &= o(dt) \quad \text{if } m \neq -1, 0, 1
\end{aligned}$$

with initial conditions $\Pr(Z(t+dt) = 0 | Z(t) = 0) = 1 - \sum_l \rho_{l,0} dt + o(dt)$ and $\Pr(Z(t+dt) = 1 | Z(t) = 0) = \sum_l \rho_{l,0} dt + o(dt)$. Denote the intensity parameters of the Markov process as $\mu_n = n\theta_n$ and $\lambda_n = \sum_l \rho_{l,n}$ and the transition matrix as \mathbf{G} . Stationarity implies that the average flow into a state is equal to the average flow out of that state. The stationary distribution, therefore, satisfies $P' \mathbf{G} = P$ where $P' = [P_0, P_1, \dots]$. Thus

$$(1 - \lambda_0)P_0 + \mu_1 P_1 = P_0$$

$$\lambda_{n-1} P_{n-1} + (1 - \lambda_n - \mu_n) P_n + \mu_{n+1} P_{n+1} = P_n \text{ for } n \geq 1$$

and solving this system of equations recursively yields $P_n = \frac{\lambda_0 \lambda_1 \dots \lambda_{n-1}}{\mu_1 \mu_2 \dots \mu_n} P_0$ for $n > 0$. For existence the probability mass function must sum up to one, that is

$$\sum_{n=0}^{\infty} P_n = P_0 + \sum_{n=1}^{\infty} P_n = P_0 + P_0 \sum_{n=1}^{\infty} \prod_{i=0}^{n-1} \frac{\lambda_i}{\mu_{i+1}} = 1$$

Therefore, for the above to hold it is necessary and sufficient to show that $\sum_{n=1}^{\infty} \prod_{i=0}^{n-1} \frac{\lambda_i}{\mu_{i+1}} < \infty$. From Proposition 5 $\lim_{n \rightarrow \infty} \lambda_n = 0$ and from Proposition 6 $\lim_{n \rightarrow \infty} \mu_n = \infty$. Applying, for instance, the ratio test it is immediate that the sequence converges (very rapidly, in fact).

Then,

$$P_0 = \left(1 + \sum_{n=1}^{\infty} \prod_{i=0}^{n-1} \frac{\lambda_i}{\mu_{i+1}} \right)^{-1}$$

Uniqueness follows from the facts that the transition probability matrix is standard and the chain is irreducible. (Grimmett and Stirzaker 2001, Theorem 6.9.21) The chain is standard because $\lim_{dt \rightarrow 0} \Pr(Z(t + dt) = i | Z(t) = j) = 1$ if $i = j$ and zero otherwise as it is clear from the definition of the transition probabilities in (B.7). The chain is clearly irreducible because any state can be visited from any state with strictly positive probability.

Furthermore, $E(n) = \sum_{n=1}^{\infty} nP_n < \infty$ because applying for example the ratio test again

$$\lim_{n \rightarrow \infty} \frac{(n+1)P_{n+1}}{nP_n} = \lim_{n \rightarrow \infty} \frac{(n+1)\lambda_n}{n\mu_{n+1}} = \lim_{n \rightarrow \infty} \frac{\lambda_n}{n\theta_{n+1}} = 0$$

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